Analysis

A wave equation for the membrane stress in a thin, infinitely long, circular, elastic shell subjected to a transient surface pressure is presented in Ref. 3. The circumferential membrane stress σ is governed by

$$\sigma'' - \sigma - \ddot{\sigma} = (a/h)p(\theta,\tau) \quad (1)$$

in which primes denote θ derivatives and dots denote differentiation with respect to the dimensionless time variable τ given by

$$\tau = ct/a, c = [E/\rho(1 - \nu^2)]^{1/2} \quad (2)$$

The angular position θ, the shell radius a, and the shell thickness h are shown in Fig. 1; the applied pressure p is measured positive radially inward; E, ν, and ρ are Young's modulus, Poisson's ratio, and density; c is the speed at which disturbances propagate in the shell; and t is time.

The membrane stress is obtained by following the procedure outlined in Ref. 3. A solution for σ is taken in the form

$$\sigma(\theta,\tau) = \sum_{n=-\infty}^{+\infty} \psi(\theta + 2n\pi,\tau) \quad (3)$$

This form of solution extends the range of θ to the interval $-\infty < \theta < +\infty$ and permits the use of the Fourier integral over the angular variable θ.

For a circumferentially moving line load traveling from $\theta = -\pi/2$ to $\theta = \pi/2$ which varies as $P(\theta)$ and travels at a constant velocity V, Eq. (1) becomes

$$\psi'' - \psi - \ddot{\psi} = \sigma_1 f(\theta)\delta(\tau - \alpha\theta) \quad (4a)$$

$$f(\theta) = \begin{cases} P(\theta)/P_0 & \text{for } |\theta| \leq \pi/2 \\ 0 & \text{for } |\theta| > \pi/2 \end{cases} \quad (4b)$$

$$\sigma_1 = \alpha P_0/h, \alpha = c/V \quad (4c)$$

where $P(\theta)$ is the magnitude of the line load, P_0 is a characteristic magnitude, and $\delta(\tau)$ is the Dirac delta function. The solution for ψ is obtained by employing the method outlined in Ref. 3, and

$$\frac{\psi(\theta,\tau)}{\sigma_1} = \frac{-1}{2} \int_0^{\pi} f(\eta)H[\tau - \alpha\eta - |\theta - \eta|]J_0\{[(\tau - \alpha\eta)^2 - (\eta - \theta)^2]^{1/2}\}d\eta \quad (5)$$

where $H(\tau)$ is the Heaviside function and J_0 is a Bessel function of the first kind of order zero. The membrane stress can then be calculated from Eqs. (3) and (5) when $f(\theta)$ is specified.

Discussion and Numerical Results

The accuracy of the impulse simulation technique described in Ref. 4 can be examined from the data presented in Fig. 2. The dashed line in Fig. 2 is the response at $\theta = 0$ produced by a simultaneously applied, impulsive load whose magnitude is $I \cos\theta$ over $|\theta| < \pi/2$ and zero over $\pi/2 < |\theta| < \pi$; whereas the solid line is the response produced by a force moving from $\theta = -\pi/2$ to $\theta = +\pi/2$ for $\alpha = 0.73$ and $P(\theta) = P_0 \sin(\pi/2 + \theta)$ shifted in time by the factor $\alpha\pi/4$.

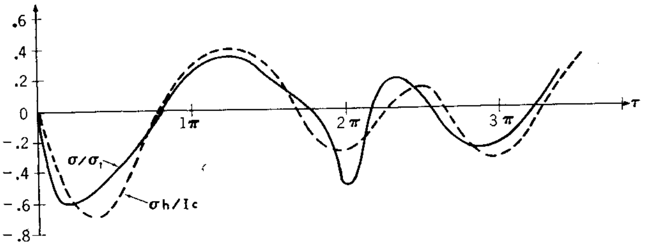

Fig. 2 Membrane stress at $\theta = 0$.

With this time shift there is close agreement between both response curves. Similar comparisons of these response data for $\theta = \pm\pi/2$ and $\theta = \pi$ with the same time shift for the moving load response show similar agreement and are available from the authors. The total impulse imparted to the cylinder for each loading in Fig. 2 is equal, and $\alpha = 0.73$ corresponds to the ratio of the bar velocity of a steel ring to the detonation velocity of the sheet explosive EL-506D.

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Shock Slip Analysis of Merged Layer Stagnation Point Air Ionization

JOHN C. ADAMS JR.*

ARO Inc., Arnold Air Force Station, Tenn.

Introduction

ANALYTICAL analysis of high-speed, high-altitude flow about a blunt body is complicated by the presence of chemical nonequilibrium phenomena in conjunction with rarefaction effects, which creates a chemically reacting, fully viscous, merged shock-layer flowfield. Most prior investigations of this problem (e.g., Refs. 1 and 2) have made the assumption that the species conservation equations can be uncoupled from the fluid-dynamic conservation equations so that previously obtained frozen-flow solutions can be used to perform the nonequilibrium species composition calculations. A recent paper by Dellinger[3] suggested that such an approximate calculation may not be strictly correct for prediction of merged-layer ionization. In addition, various assumptions regarding the chemical and diffusion models often are made with little or no attention given to their effect on the resultant solution. The present Note is an attempt to clarify the effects of reaction rates, species diffusion, and coupled vs decoupled species concentration calculations on merged-layer air ionization predictions using a nonequilibrium thin viscous shock-layer analysis for comparison with the results of Refs. 1–3.

Analysis

The reader is referred to Refs. 4 and 5 for complete details of the analytical method used in the present work. Basically,

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* Supervisor, Theoretical Gas Dynamics Section, Hypervelocity Branch, Aerophysics Division, von Kármán Gas Dynamics Facility. Member AIAA.

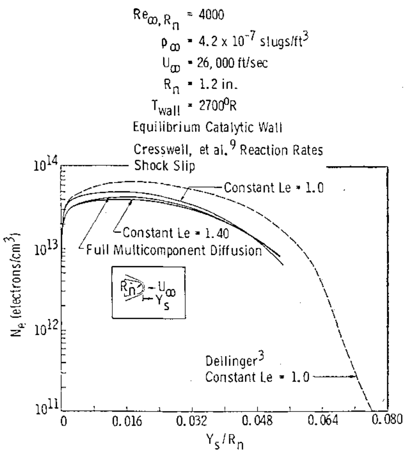

Fig. 1 Stagnation-point electron-density predictions showing effects of diffusion model.

the analysis is that of a stagnation point thin viscous shock layer for seven chemical species (O, N, O_2, N_2, NO, NO^+, e^-) in chemical nonequilibrium, including the centrifugal effects attributable to a normal pressure gradient across the layer. Between the shock (taken to be chemically frozen) and the body, the flow is considered to be in chemical nonequilibrium with the shock standoff distance determined as part of the solution. Across the shock, the static pressure and normal velocity component are discontinuous according to the usual Rankine-Hugoniot conditions; in addition, so-called shock-slip effects (discontinuities in species composition, tangential velocity, and total enthalpy) are allowed immediately behind the shock following Cheng.[6] These shock-slip effects allow for merging of the viscous layer with the shock and effectively account for the effects of diffusion, heat transfer, and shear stress at the shock. The actual numerical solution is an iterative implicit finite-difference technique following Blottner[7] in which velocity, temperature, and species concentrations are solved for simultaneously. The present analysis differs from that of Lee and Zierten[1] and Dellinger[3] (who actually integrated through the shock structure allowing for chemical reactions) in that here the shock transition region is treated as chemically frozen in deriving the shock-slip edge conditions so that the actual details of the shock structure are not considered; see Cheng[6] for a complete discussion of this point.

Discussion of Results

Shown in Fig. 1 are stagnation-point electron-density predictions for the $Re_\infty = 4000$ flow condition of Dellinger[3] using an equilibrium catalytic wall in conjunction with the chemical reaction system and reaction rates of Ref. 3. In order to assess the effects of diffusion models, multicomponent diffusion vs constant Lewis-Semenov number results are presented; also shown for comparison purposes is the prediction of Dellinger[3] who used a constant Lewis-Semenov number of unity. The choice of diffusion model has little effect on the resulting electron-density prediction. Furthermore, the shock-slip approach taken in the present analysis appears to adequately describe electron density as compared to the results of Dellinger.[3]

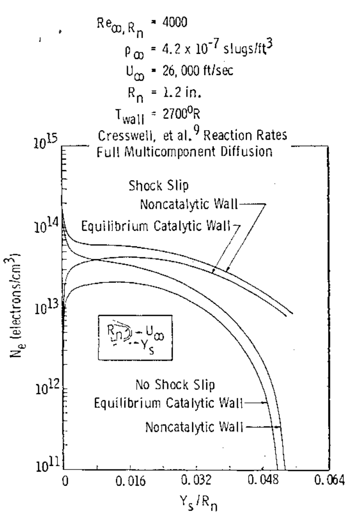

Fig. 2 Stagnation-point electron-density predictions showing effects of shock conditions and wall-catalytic condition.

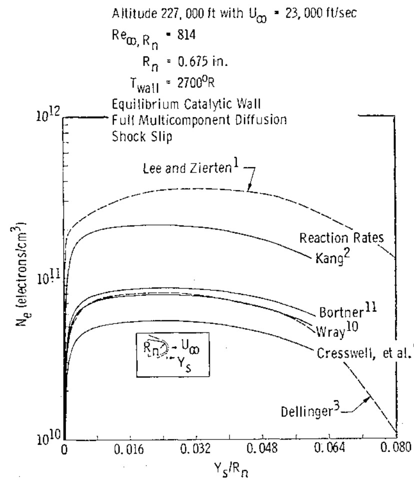

Fig. 3 Stagnation-point electron-density predictions showing effects of reaction rates.

A previous Note by Adams[5] employed a nonequilibrium thin viscous shock-layer model with all shock-boundary conditions taken from Rankine-Hugoniot jump conditions for a chemically frozen real gas; i.e., no shock-slip effects were considered. Figure 2 presents results for the same flow conditions defined previously using both shock-slip and no shock-slip models. Both equilibrium catalytic and noncatalytic surface conditions are included. Full multicomponent diffusion in conjunction with the previously defined reaction system and rates is used. The large differences near the outer edge of the shock layer clearly indicate the necessity of including shock-slip effects for such flow conditions. Furthermore, peak electron density is underpredicted by a factor of two in the equilibrium catalytic-wall case using the no shock-slip model. The influence of the wall-catalytic condition can be seen to extend over the entire shock layer with the principal effect concentrated in the near wall region. Note also that the shock-layer thickness increases as the surface is changed from catalytic to noncatalytic. Such is due to the density of the gas mixture being larger near the catalytic surface and is in agreement with the findings of Chung et al.[8]

In order to assess the effects of different reaction rates on the resultant electron-density prediction, a systematic study is required. In Fig. 3 four different sets of reaction rates are applied to the $Re_\infty = 814$ flow condition of Lee and Zierten[1] using the present shock-slip model. Also shown in Fig. 3 are the predictions of Dellinger[3] (who used the rates of Cresswell et al.[9]) and Lee and Zierten[1] (who used the rates of Wray[10] in conjunction with the frozen-flow solution of Chung et al.[8]) Kang[2] reports good agreement with Lee and Zierten,[1] and hence this prediction is not shown plotted on Fig. 3. A factor of four maximum difference in peak-electron density due to the different reaction rates is observed from the shock-slip results. The agreement between Dellinger[3] and the present shock-slip solution using Wray's rates has no relevance since Dellinger used the rates of Cresswell et al.[9] and it has previously been shown in Fig. 1 that the prediction of Dellinger is slightly higher than the present shock-slip result using common reaction rates. Based on Fig. 3, it becomes obvious that the large difference between Lee and Zierten[1] and Dellinger[3] cannot be attributed solely to differences in reaction rates. Hence, the method of solution (coupled vs decoupled) must be important as suggested by Dellinger[3] in his paper.

Conclusions

Nonequilibrium, multicomponent air calculations for electron-density distributions in the stagnation point, merged viscous shock layer have been presented using a shock-slip analysis in conjunction with a coupled, iterative finite-difference numerical solution. The present study yielded four main conclusions summarized as follows: 1) Including of shock-slip effects are necessary for analysis of merged viscous-layer electron density using a thin viscous shock-layer model. Such a shock-slip approach appears to be valid based on comparison with solutions which include details of the shock structure. 2) Predicted electron-density distributions are relatively insensitive to choice of diffusion model (constant Lewis-Semenov number for all species vs full multicomponent diffusion). 3) For an equilibrium catalytic-wall condition, a factor of four maximum difference in peak electron density resulted from the four different sets of reaction rates used in the present study. The catalytic condition of the wall influences the entire shock layer with the principal effect concentrated in the near-wall region. 4) Decoupled calculations in which the species concentrations are solved for using frozen-flow fluid-dynamic solutions appear to overpredict the electron density as compared to coupled calculations such as that of Dellinger[3] and the present work.

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Acoustic Tracking of Supersonic Objects

Walter P. Reid*
U.S. Naval Ordnance Laboratory, Silver Spring, Md.

Introduction

In this Note equations are developed for determining the location, speed, and direction of motion of an object traveling faster than the speed of sound from the times at which the shock wave arrives at some microphones.

Solutions to this problem have been given previously by Zaroodny[1] and Reid.[2] However, the method of Ref. 2 was found to be impractical to use because it was too sensitive to small errors in the locations of the microphones and/or in the recorded times at which the shock wave arrived at the microphones. The purpose of the present Note is to give a calculation procedure in which acceptable accuracy is more easily obtained. In Ref. 2, all the details about the missile path were determined from a single cluster of microphones. In this Note, less information is obtained from a cluster of microphones, and three or more of them are used. They may be widely separated. This in itself results in a significant improvement in the accuracy of tracking when there are small errors in the data. In addition, the present mathematical analysis permits extra readings to be used easily. With exactly three clusters of microphones, Eqs. (2) and (4) below, each give three equations in three unknowns and serve to determine the missile's velocity and location. If more than three clusters of microphones are used, more than three equations will be obtained from Eqs. (2) and (4), and the method of least squares can be used to obtain best values for the calculated velocity and location.

Analysis

Assume that an object is traveling in a straight line in a fluid with a constant speed v greater than the speed of sound in the medium, and that it generates a shock wave that is a right circular cone except perhaps in the neighborhood of the object. Let it be desired to find the speed and line of flight of the vertex of the cone, and hence of the object, from the times t at which the shock wave arrives at various microphones.

Let three lines of microphones be placed so as to pass through a point with the lines parallel respectively to the x, y, and z axes. This cluster of microphones will be a listening station. Use the microphones to find the time at which the shock cone arrives at the intersection of the rows of microphones, and also the partial derivatives of t with respect to x, y, and z at that point, and therefore ∇t.

∇t is a vector perpendicular to the surface t = const, and hence to the shock cone. Since a shock wave moves in the direction of ∇t with the speed of sound c it follows that $|\nabla t| = 1/c$, or

$$(\partial t/\partial x)^2 + (\partial t/\partial y)^2 + (\partial t/\partial z)^2 = 1/c^2 \tag{1}$$

Thus, the partial derivatives of t at a point are dependent. One of them may be calculated from the other two, except for sign, so that three lines of microphones are not needed at a listening station, but only two.

The object moves a distance vT along the axis of the cone in time T while the shock wave is traveling a distance cT perpendicular to itself, and hence perpendicular to an element of the cone. Thus, the cosine of the angle between $\mathbf{v}$ and ∇t is c/v. Therefore

$$\mathbf{v}\cdot\nabla t = 1 \tag{2}$$

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* Research Mathematician, Mathematics Department.